Mathematics Specialist Units 1 and 2 Formula Sheet

Trigonometry

$\sec(\theta) = \dfrac{1}{\cos\theta}$, $\cos\theta \neq 0$

$\operatorname{cosec}(\theta) = \dfrac{1}{\sin\theta}$, $\sin\theta \neq 0$

$\cot(\theta) = \dfrac{1}{\tan\theta}$, $\tan\theta \neq 0$

$\sin^2\theta + \cos^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$

$\sin 2A = 2\sin A\cos A$

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A\\ &= 2\cos^2 A - 1\\ &= 1 - 2\sin^2 A\end{aligned}$$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

$\cos A\cos B = \dfrac{1}{2}[\cos(A - B) + \cos(A + B)]$

$\sin A\sin B = \dfrac{1}{2}[\cos(A - B) - \cos(A + B)]$

$\sin A\cos B = \dfrac{1}{2}[\sin(A + B) + \sin(A - B)]$

$\cos A\sin B = \dfrac{1}{2}[\sin(A + B) - \sin(A - B)]$

In any triangle *ABC*

Sine rule

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule

$a^2 = b^2 + c^2 - 2bc\cos A$

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

Vectors

Magnitude

$|\boldsymbol{a}| = |(a_1, a_2)| = \sqrt{{a_1}^2 + {a_2}^2}$

Unit vector

$\hat{\boldsymbol{a}} = \dfrac{1}{|\boldsymbol{a}|}\cdot\boldsymbol{a}$

Scalar product

$\boldsymbol{a}\cdot\boldsymbol{b} = |\boldsymbol{a}|\cdot|\boldsymbol{b}|\cos\theta = a_1b_1 + a_2b_2$

Vector projection of **a** on **b**

$\boldsymbol{p} = p\cdot\hat{\boldsymbol{b}}$ where $p = \boldsymbol{a}\cdot\hat{\boldsymbol{b}}$

Index laws

For $a, b > 0$ and m, n real,

$a^m a^n = a^{m+n}$

$\dfrac{a^m}{a^n} = a^{m-n}$

$(a^m)^n = a^{mn}$

$a^m b^m = (ab)^m$

$a^0 = 1$

$a^{-m} = \dfrac{1}{a^m}$

For $a > 0$ and m an integer and n a positive integer,

$a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

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Function

If $ax^2 + bx + c = 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Number

Natural

$\mathbb{N} = \{1, 2, 3, 4, \dots\}$

Integer

$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$

Rational

$\mathbb{Q} = \{x: x = \frac{a}{b}$, where $a, b \in \mathbb{Z}, b \neq 0\}$

Real

The set of real numbers $\mathbb{R}$ consists of the set of all rational and irrational numbers.

Complex

$\mathbb{C} = \{z: z = a + bi$, where $a, b \in \mathbb{R}, i^2 = -1\}$

$\bar{z} = a - bi$

Combinatorics

There are $n(n-1)(n-2) \times \dots \times 3 \times 2 \times 1 = n!$ ways to arrange n objects in an ordered list.

Permutations

$$\begin{aligned} {}^nP_r &= n(n-1)(n-2)\dots(n-r+1) \\ &= \frac{n!}{(n-r)!} \end{aligned}$$

Combinations

$$\begin{aligned} {}^nC_r &= \binom{n}{r} \\ &= \frac{n(n-1)(n-2)\dots(n-r+1)}{r!} \\ &= \frac{n!}{r!\,(n-r)!} \end{aligned}$$

Inclusion-exclusion principle

$$\begin{aligned} n(A \cup B \cup C) = {} & n(A) + n(B) + n(C) \\ & -n(A \cap B) - n(A \cap C) - n(B \cap C) \\ & +n(A \cap B \cap C) \end{aligned}$$

Matrices

Determinant

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $\det A = ad - bc$

Inverse

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Dilation

$$\begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

Rotation

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Reflection

$$\begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix}$$

Shear

$\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$ or $\begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix}$